Acoustic for Underwater Sensing

Time and Frequency Modelling

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Introduction to advanced marine technologies

SUMMER SCHOOL (SS1)
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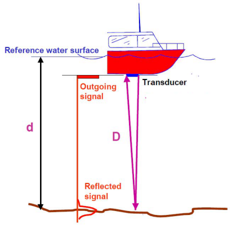
Summary

- Basic concepts of signal processing applied to Underwater acoustics
- The correlation (travel time/ range estimation)
- Linear systems (impulse response and frequency response)
- Time $\leftrightarrow$ Frequency modelling

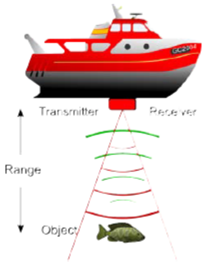
The echosounder[1]

- Object detection
- Range estimation

fathometer (water depth)

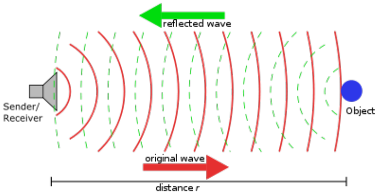


fishfinder



[1] pictures from wikipedia

Principle of range estimation



1. A short pulse is transmitted
2. The pulse is reflected by the object (reflector)
3. The receiver detects the reflected pulse (echo)

- travel time (τ): the period elapsed from the transmission till detection of echo at the receiver
- sound speed in water(c)
- reflector's range

$$r = \frac{c\tau}{2}$$

Data model

- $s(t)$ transmitted (pulse) signal,
- τ travel time (time of flight),
- a attenuation weight, transmission loss and reflection loss,
- $n(t)$ noise,
- $y(t)$ received signal.

$$y(t) = as(t-\tau) + n(t)$$

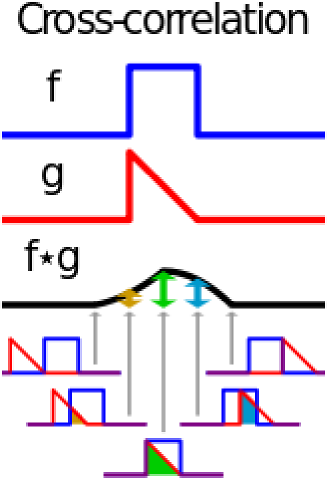
Questions:

- How to select the signal $s(t)$?
- How to detect the signal?, i.e., How to estimate τ?

The **cross-correlation** $r_{fg}(t)$ between signals $f(t)$ and $g(t)$ is a measure of similarity between them as a function of a delay (time-lag),

$$\begin{aligned} r_{fg}(t) &= f(t) \star g(t) \\ &= \int_{-\infty}^{\infty} f(\tau)g(\tau - t)\mathrm{d}\tau \\ &= \int_{-\infty}^{\infty} f(t+\tau)g(\tau)\mathrm{d}\tau \end{aligned}$$

- $r_{gf}(t) = r_{fg}(-t)$ (noncommutative)



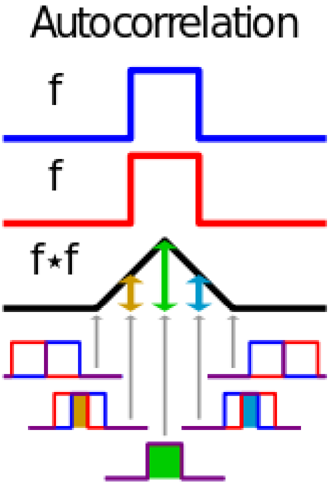
Signal autocorrelation

The **Autocorrelation** $r_{ff}(t)$ is the cross-correlation of a signal with itself.

- $r_{ff}(0)$ represents the energy contents of a signal

$$r_{ff}(0) = \int_{-\infty}^{\infty} |f(\tau)|^2 \mathrm{d}\tau$$

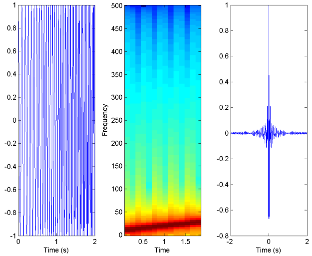
- $r_{ff}(t) < r_{ff}(0)$, when $t \neq 0$

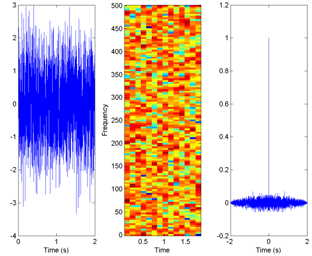


Examples of autocorrelation

chirp
frequency band 10-20Hz,
duration 2s

noise sample
white Gaussian
mean (μ) 0, variance ($\sigma^2 = 1$)





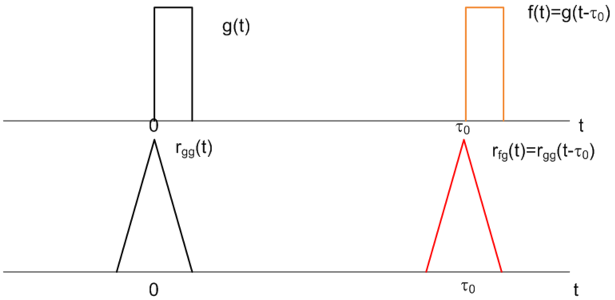
- What is the autocorrelation of a sinusoid (ton)?
- Autocorrelation (peak) width is inverse to signal's frequency bandwidth

Properties(1)

- when $f(t)$ is $g(t)$ delayed (τ_0 is the delay) , i.e $f(t) = g(t - \tau 0)$,

$$r_{fg}(t) = r_{gg}(t - \tau_0)$$

The cross-correlation between $g(t)$ and $f(t)$ is the delayed autocorrelation of $g(t)$, i.e. $r_{gg}(t)$ centered at τ_0



Properties(2)

- when $f(t) = ag(t)$, where a is a gain (or attenuation), then

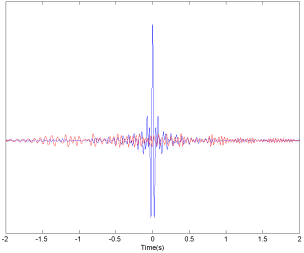
$$r_{fg}(t) = ar_{gg}(t)$$

- the correlation is additive, i.e. the cross-correlation between $f(t) = u(t) + v(t)$ and $g(t)$ is equal to the sum of the cross-correlation between $u(t)$ and $g(t)$ with the cross-correlation between $v(t)$ and $g(t)$

$$r_{gf}(t) = r_{gu}(t) + r_{gv}(t)$$

Cross-correlation between signal and noise

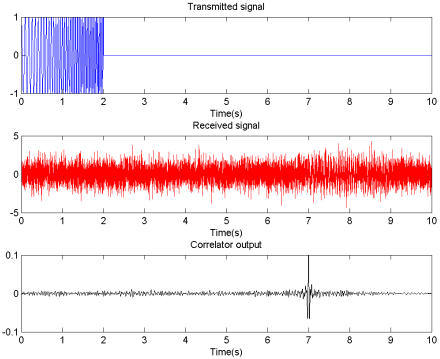
Chirp's autocorrelation (blue), cross-correlation between chirp and zero mean white Gaussian noise (red)



- Noise is a random signal, therefore we should consider the average correlation over various noise realizations.
- Assuming that signal and noise are independent the average correlation is zero.

Travel time estimator

- the autocorrelation of the probe (transmitted) signal $s(t)$ should be narrow (well defined peak)
- perform the cross-correlation between the probe and the received signal $y(t) = as(t - \tau) + n(t)$



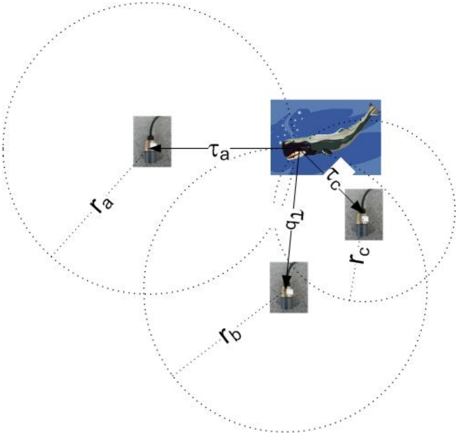
Travel time estimator

$$\tau = \arg \left\{ \max |r_{ys}(t)|^2 \right\}$$

Once τ is estimated, assuming that c (sound speed) is known, the object range estimation is straightforward.

Passive localization

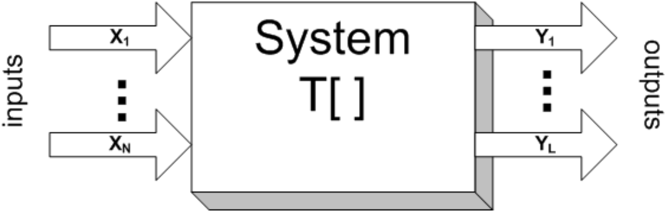
- The target transmits broadband signals
- The receivers are synchronized
- The signals received in various receivers are cross-correlated



- $\tau_{ab} = \arg\left\{\max|\mathrm{r_{ab}(t)}|^2\right\}, \ldots$
- $\tau_{ab} = \tau_a - \tau_b$
  $\tau_{ac} = \tau_a - \tau_c$
  $\tau_{bc} = \tau_b - \tau_c$
- $\tau_{ab}c = r_a - r_b$
  $\tau_{ac}c = r_a - r_c$
  $\tau_{bc}c = r_b - r_c$
- solve the linear system to estimate r_a, r_b, r_c

System

- A system is a transformation (mapping) of inputs into outputs



$$[y_1(t), y_2(t), ..., y_L(t)] = T\,[x_1(t), x_2(t), ..., x_N(t)]$$

- Single Input Single Output systems(SISO).

$$y(t) = T[x(t)]$$

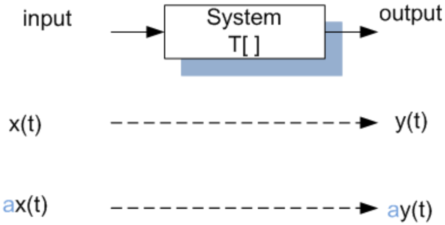
  - $y(t)$ is the **system response** to system input $x(t)$

Linear system (1)

Let $y(t)$ the response of the system to the input $x(t)$:

- Linear systems obey the **principle of superposition** (linearity):
  - Scaled

$$T[ax(t)] = aT[x(t)]$$

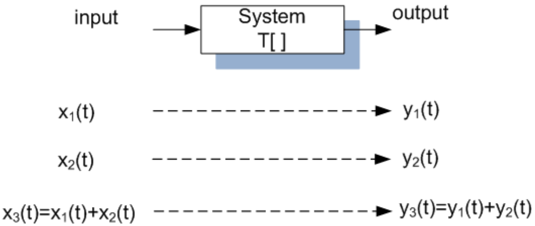


Linear system (2)

Let $y(t)$ the response of the system to the input $x(t)$:

- Linear systems obey the **principle of superposition** (linearity):
  - Summed

$$T[x_1(t) + x_2(t)] = T[x_1(t)] + T[x_2(t)]$$

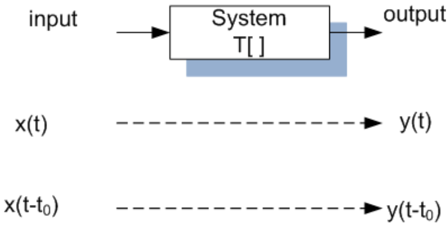


- Combining both:

$$T[ax_1(t) + bx_2(t)] = aT[x_1(t)] + bT[x_2(t)]$$

Time invariant system

- A system is **time invariant** if:
  - whether it applies an input to the system now or t_0 seconds later, the output will be identical except for a time delay of t_0
  - $$y(t) = T[x(t)] \Longrightarrow y(t-t_0) = T[x(t-t_0)], \forall t_0 \in \mathbb{Z}$$



Impulse response

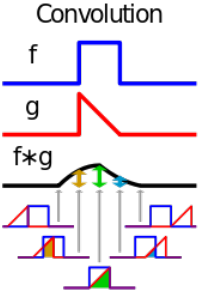
- The **impulse response** of a system is its output when the input is an (unit) impulse (dirac function).

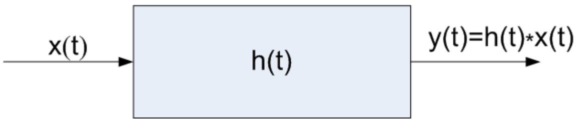
- The impulse response characterize the system.
- For a **linear time-invariant (LTI) system** with impulse response $h(t)$ and input $x(t)$, the output is given by the convolution.

Convolution & Impulse response

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)\mathrm{d}\tau = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\mathrm{d}\tau = \mathbf{h(t)} * \mathbf{x(t)}$$

(convolution ($*$) is commutative)



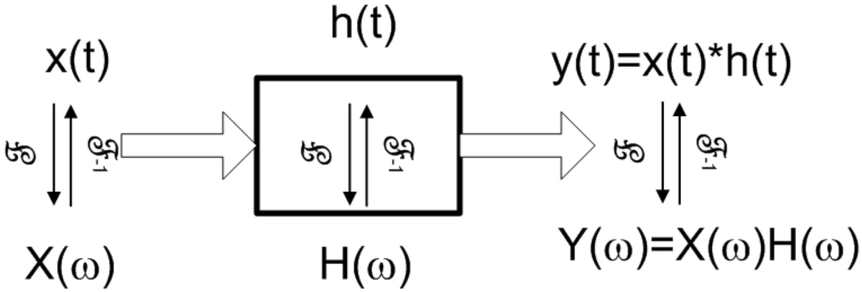


Frequency response

- The frequency response of a system determines, for a sinusoidal input at a given frequency, the gain and phase shift introduced by the system. For LTI systems:
  - If the input is a sinusoid, the output is also a sinusoid at the same frequency (the amplitude and phase depends on the frequency response)
  - The frequency response $H(\omega)$ is the Fourier transform of the impulse response $h(t)$

time (t) $\longrightarrow$ freq. (ω) $$H(\omega) = \mathcal{F}\{h(t)\} = \int_{-\infty}^{+\infty} h(t) e^{-j\omega t} \mathrm{d}t$$

freq. (ω) $\longrightarrow$ time (t) $$h(t) = \mathcal{F}^{-1}\{H(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(\omega) e^{j\omega t} \mathrm{d}\omega$$

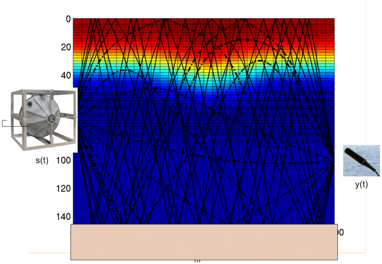


$$H(\omega)=Y(\omega)/X(\omega)$$

$X(\omega), Y(\omega), H(\omega)$, $\mathcal{F}$(FT) of x(t), y(t), h(t)

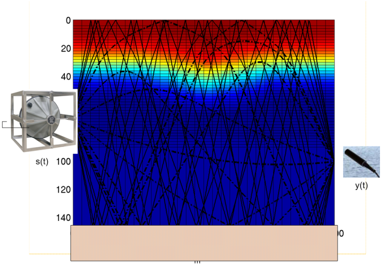
Multipath

- Due to refraction and reflections the waveform transmitted by the source is received as a sum of various echoes.



- Each echo is characterized by the propagation path (trajectory), attenuation and travel time,
  - it depends also on environmental characteristics of the ocean (underwater acoustic channel).

Impulse response of an underwater channel



N, A_n, τ_n characterize the underwater channel between the transmitter and the receiver

- The impulse response is given by:

$$h(t) = \sum_{n=1}^{N} A_n \delta(t - \tau_n)$$

N number of echoes; A_n, τ_n, n^{th} echo attenuation and travel time (delay).

- The received signal $y(t)$ is given by:

$$y(t) = \sum_{n=1}^{N} A_n s(t - \tau_n) + n(t)$$

where $s(t)$ is the transmitted waveform and $n(t)$ is the noise.

Arrival pattern (1)

How to estimate N, A_n, τ_n ?

- Problem of amplitude and time delay estimation of a known signal in noise.
- A possible solution is cross-correlating the received signal $y(t)$ with the transmitted waveform $s(t)$.

Arrival pattern (2)

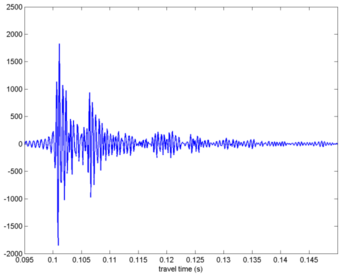
- The received signal $y(t)$ is cross-correlated with transmitted waveform $s(t)$:

$$
\begin{aligned}
E\{y(t) \star s(t)\} &= (using\ correlation) \\
E\{y(t) * s^H(-t)\} &= (using\ convolution) \\
&E\{[s(t) * h(t) + n(t)] * s^H(-t)\} \\
&E\{\underbrace{[s(t) * s^H(-t)]}_{r_{ss}} * h(t)\} + E\{\underbrace{n(t) * s^H(-t)}_{\approx 0}\}
\end{aligned}
$$

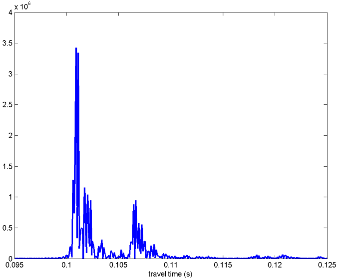
$$
\begin{aligned}
r_{ys}(t) &= r_{ss}(t) * h(t) \\
&= \sum_{n=1}^{N} A_n r_{ss}(t - \tau_n),
\end{aligned}
$$

- the output is a (time) function with N peaks $A_1, \ldots A_n$ occurring at delays $\tau_1, \ldots \tau_N$

Arrival pattern (3)



Cross-correlation



Arrival pattern

- Arrival pattern

$$a(t) = |r_{ys}(t)|^2.$$

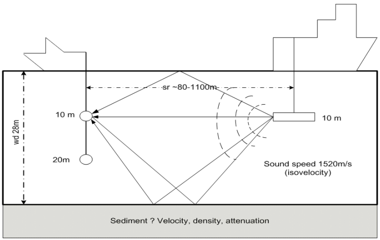
- Very often the amplitudes are represented in decibel (dB-logarithmic scale)

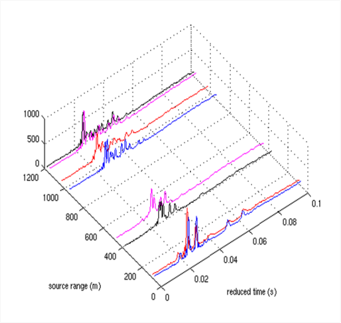
$$r(t) = 10log(a(t))$$

- Emphasizes latter (weaker) arrivals

Example: Influence of source location

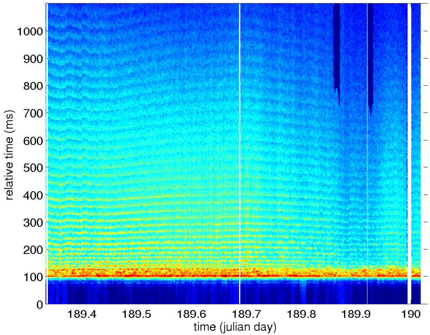
- OAEx (July 2009), Ilha dos porcos, Arraial do Cabo, Brazil
- sonar source (3-4kHz),
- source range: 50 m to 1200 m,
- 2 hydrophones: 10 m and 20 m depth.

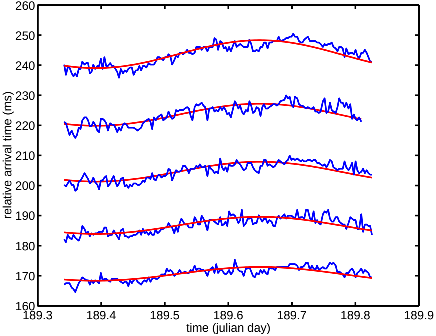




Example: Surface tide (water depth)

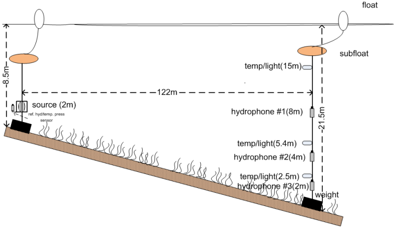
- INTIMATE'98 data set (Gulf of Biscay)
- shallow water (wd 140m, sr 10km)

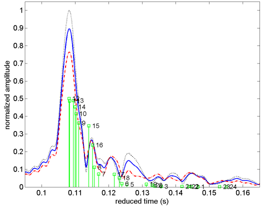


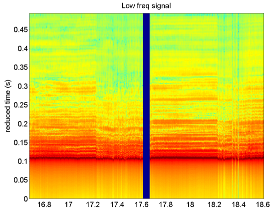


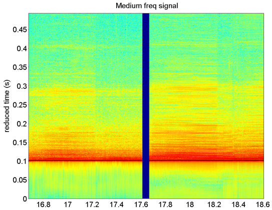
Example: physic-chemical parameters (O_2)

- transmissions over a seagrass bed (*Posidonia ocenica*), Bay of La Revellata, Corsica
- attenuation related to photosynthetic activity (released O_2 bubbles and air in aerenchyma)

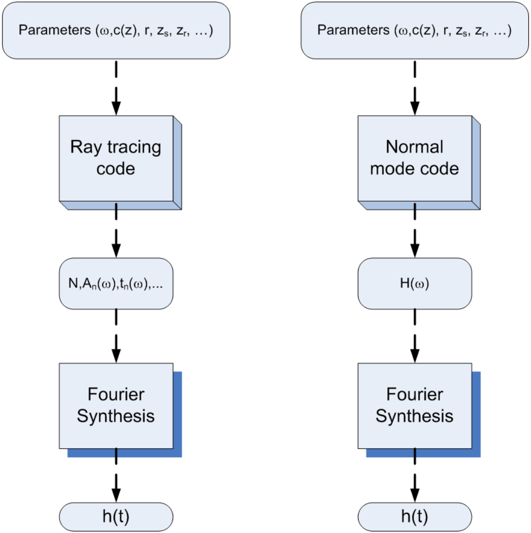








Broadband modeling (time domain modeling)



Fourier synthesis

- $H(\omega)$ is given by the model
- impulse response

$$h(t) = \int_{-\infty}^{+\infty} H(\omega) e^{j\omega t} \mathrm{d}\omega$$

- received signal $y(t) = s(t) * h(t)$

$$y(t) = \int_{-\infty}^{+\infty} S(\omega) H(\omega) e^{j\omega t} \mathrm{d}\omega$$

  $S(\omega)$ source's frequency response
- cross-correlation $r_{ys}(t) = r_{ss}(t) * h(t)$

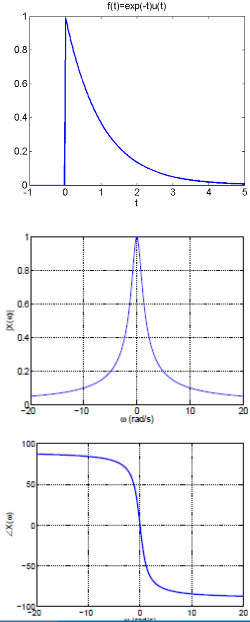
$$r_{ys}(t) = \int_{-\infty}^{+\infty} S(\omega) S^H(\omega) H(\omega) e^{j\omega t} \mathrm{d}\omega$$

FT of real signals

If $x(t)$ is a real signal

$$X(-\omega) = X^H(\omega)$$

- the module $|X(\omega)|$ is an even function
- the argument (phase) $\angle X(\omega)$ is an odd function
- Model computes $H(\omega)$ only for positive frequencies



Practical issues (1)

- select f_s depending on the max frequency of source signal
- select T_0 depending on the expected time spread of $h(t)$
- determine N ($N \geq \text{round}\{T_0/f_s + 1\}$)
- determine frequency step $\Delta f = f_s/N$
- determine the discrete frequencies (frequency indexes) for the signal's band
- determine $H(\omega)$ (only "positive" frequencies) using a propagation model

Practical issues (2)

- Normal modes: for the selected frequencies compute $H(\omega)$
- Ray tracing:
  - compute A, τ for the rays set
  - for the central frequency (or selected frequencies) compute $H(\omega)$ using the time-delay property of FT $\mathcal{F}\{x(t-t_0)\} = X(\omega)e^{-j\omega t_0}$

$$H(\omega) = \sum_{l=1}^{L} A_l e^{-j\omega\tau_l}, L \text{ number of rays}$$

- using the FT property of real signals determine the complementary part of $H(\omega)$
- using inverse DFT (IFFT) compute $h(t)$
  - to compute $y(t)$ use $S(\omega)H(\omega)$ (in place of $H(\omega)$)
  - to compute $r_{ys}(t)$ use $S(\omega)S^H(\omega)H(\omega)$
  - $S(\omega)$ is the source spectrum

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Thank you for your attention!

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