Acoustic for Underwater Sensing

Array Data Processing - The beamformer

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Introduction to advanced marine technologies

SUMMER SCHOOL (SS1)
Almada, Portugal

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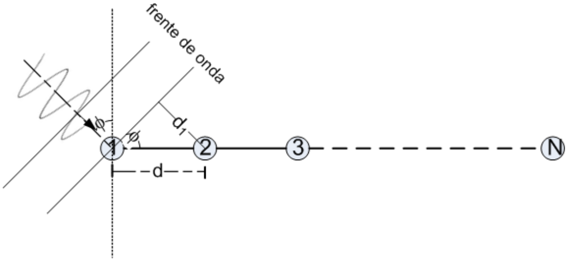
Outline

- Uniform linear array
- The principle of beamforming
- Time domain beamformer
- Frequency domain beamformer
- Beamformer issues
- Beamformer Examples
- Generalized beamformer
- Matched field processing

The hydrophones' array

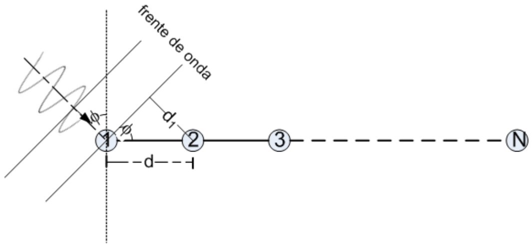
Uniform linear array: identical sensor elements, aligned along a straight line with equal element spacing.

- N hydrophones, d element spacing.
- a plane wave impinges on the array at direction Φ



- the sound speed is c
- the sinusoidal wave of frequency $\omega_0 = 2\pi f_0$,
  - wavelength $\lambda = c/f_0$, wavenumber $k = \omega_0/c = 2\pi/\lambda$

Signal sampled by hydrophone arrays (1)



- Assuming hydrophone 1 as reference ($\tau_1 = 0$)
  - $\tau_2 = d \sin(\Phi)/c$ is the signal delay at hydrophone 2,
  - $\tau_i = (i-1) d \sin(\Phi)/c$ is the delay at hydrophone i.
- The signal at the several hydrophones of the array is:

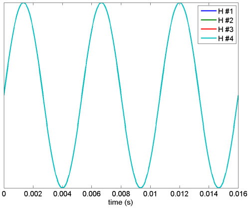
$$y_1(t) = s(t), \qquad \tau_1 = 0$$
$$y_2(t) = s(t - \tau_2)$$
$$\ldots$$
$$y_N(t) = s(t - \tau_N)$$

where $s(t)$ is the signal at hydrophone 1.

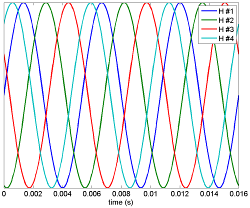
Signal sampled by hydrophone arrays (2)

- Example: 4 element array, sinusoid
  $f = 187.5\,Hz, \;\; d = 4\,m, \;\; c = 1500\,m/s, \Phi = 0°, 35°, 90°$

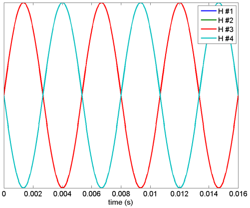
$0°$



$35°$



$90°$



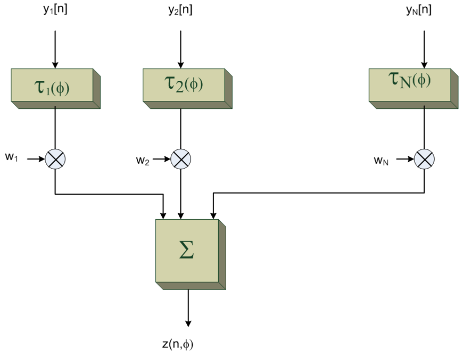
How to sum coherently (in phase) the signal at the several array elements ?

The principle of Beamforming

- Steering the array (mechanically) to wave direction, i.e. $\Phi = 0^\circ$
  - summing the hydrophones outputs gives a gain of N when compared with a single hydrophone.
- The steering operation can be achieved by signal processing techniques
  - in the time domain (time domain beamformer)
  - in the frequency domain (frequency domain beamformer)
- Beamformer applications:
  - increase the SNR (signal to noise ratio) for a given direction,
  - select the signals from a given direction,
  - direction of arrival (DOA) estimation.

Time domain beamformer (1)

- $y_1[n]$, $y_1[n]$, ... , $y_N[n]$ signals acquired at sampling frequency f_s ($t_n = n/f_s$)
- the signals at different hydrophones are **delayed** according the direction
- the delayed signals are **summed** up (The signals could also be **weighted** w_i)
- weight, delay and sum beamformer



- **How to implement the delays?**

Time domain beamformer (2)

- steering to direction Φ, the signal $y_i[n]$ at hydrophone i is delayed

$\tau_i(\Phi) = k_{\Phi i} 1/f_s$ $\quad k_{\Phi i}$ is an integer representing the n. of samples

$k_{\Phi i} = (i-1)m_\Phi$ $\quad m_\Phi$ n. of samples between neighbor hydrophones

- The beamformer output is given by

$$z(n, \Phi) = \sum_{i=1}^{N} w_i y_i[n - (i-1)m_\Phi]$$

Time delay beamformer (3)

- The delay between neighbor hydrophones is $d \sin(\Phi)/c$

$$\frac{d \sin(\Phi)}{c} = m_\Phi \frac{1}{f_s}$$
$$\sin(\Phi) = m_\Phi \frac{c}{df_s}$$

- Since $-1 \leq \sin(\Phi) \leq 1$ and m_Φ is an integer, only few directions could be steered

$$\left| m_\Phi \frac{c}{df_s} \right| \leq 1$$

  - Example: if $f_s = 750\, Hz$, $c = 1500\, m/s$, $d = 4\, m$ only $0°$, $\pm 30°$, $\pm 90°$ are steered
  - Possible solution: increase the sampling frequency using interpolation (up sampling)

Frequency domain beamformer (1)

- The delays are compensated in the frequency domain using the FT propriety:
  - a time delay gives rise to a frequency shift:

$$\mathfrak{F}\{x(t)\} = X(\omega)$$

$$\mathfrak{F}\{x(t-\tau_0)\} = X(\omega)e^{-j\omega\tau_0}, \tau_0 \text{ is the delay}$$

- Assuming a wave from direction Φ_0, at i-th hydrophone, one can write:

$$Y_i(\omega) = S(\omega)e^{-j\omega(i-1)\tau_{\Phi_0}}, \text{ where } \tau_{\Phi_0} = \frac{d\sin(\Phi_0)}{c}$$

- The beamformer output at frequency ω is given by:

$$Z(\omega,\Phi_0,\Phi) = \sum_{i=1}^{N} Y_i(\omega)e^{j\omega(i-1)\tau_\Phi} = S(\omega)\sum_{i=1}^{N} e^{-j\omega\tau_{\Phi_0}}e^{j\omega(i-1)\tau_\Phi}$$

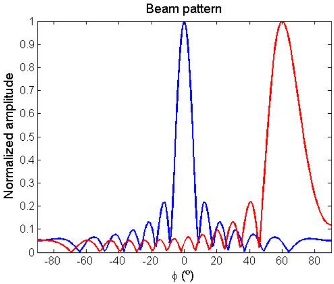
Frequency domain beamformer(2)

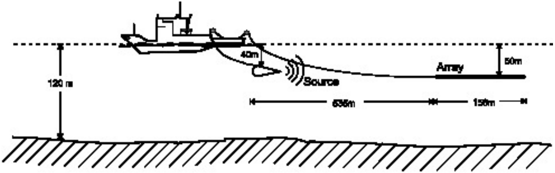
The beamformer output

$$B(\omega, \Phi_0, \Phi) = \sum_{i=1}^{N} w_i e^{-j\omega\tau_{\Phi_0}} e^{j\omega\tau_{\Phi}}$$

where $\tau_\Phi = \frac{d\sin(\Phi)}{c}$ has the maximum when $\Phi = \Phi_0$ (w_i is a weight), and the output signals are coherently summed.

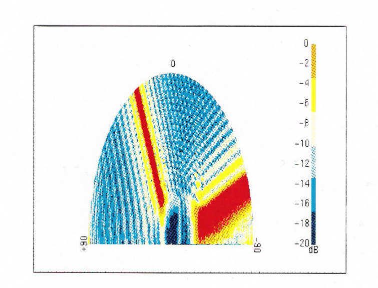
- the module of $B(\omega, \Phi_0, \Phi)$ is the Beam steering pattern, which characterizes the Beamformer.
- Example:
  $N = 20,\ f_s = 750\,Hz,$
  $f = 500\,Hz, c = 1500\,ms^{-1},$
  $\Phi_0 = 0°(blue), 60°(red).$



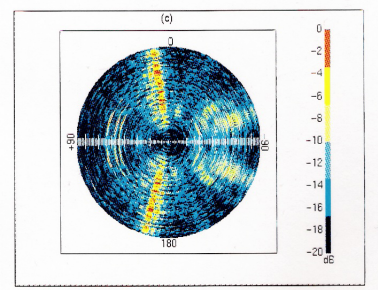


- MAST experiment, Strait of Sicily
- $N = 40,\ d = 4\,m,\ f = 25 - 187.5\,Hz$

simulation
sources at : 25° e -70°



experimental data



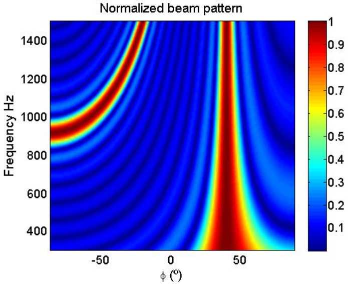
Beamformer issues

The resolution of the beam pattern is influenced by:

- the distance between each sensor;
- the frequency of the emitted signal;
- the configuration of the array;
- the number of sensors (receivers).

The distance (d) between each sensor is also related to the working frequency (f_{max}) of the array, and is provided by the Nyquist theorem:

$$d \leq \frac{\lambda}{2} = \frac{c}{2f_{max}}$$



The frequency domain beamformer is efficiently implemented using the Fast Fourier Transform (FFT).

Beamformer

- Recall the (plane-wave) beaformer output:

$$Z(\omega,\Phi) = \sum_{i=1}^{N} Y_i(\omega) e^{j\omega(i-1)\tau_\Phi}$$

- using a matrix notation (omitting ω)

$$Z(\Phi) = \mathbf{m}(\Phi)^H \mathbf{Y}$$

where $\mathbf{Y}$ is the data vector and $\mathbf{m}(\Phi)$ is the replica vector (assuming a plane-wave model)

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} \quad \mathbf{m}(\Phi) = \begin{bmatrix} 1 \\ e^{-j\omega\tau_\Phi} \\ \vdots \\ e^{-j\omega(N-1)\tau_\Phi} \end{bmatrix}$$

Beamformer

- The beam power

$$B(\Phi) = \left|\mathbf{m}(\Phi)^H \mathbf{Y}\right|^2 = \mathbf{m}(\Phi)^H \mathbf{Y}\mathbf{Y}^H \mathbf{m}(\Phi)$$

- Having K data snapshots of data available

$$B(\Phi) = \frac{1}{K} \sum_{k=1}^{K} \mathbf{m}(\Phi)^H \mathbf{Y_k} \mathbf{Y_k}^H \mathbf{m}(\Phi)$$

- or

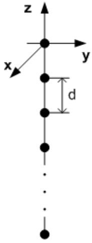
$$B(\Phi) = \mathbf{m}(\Phi)^H \left\{ \frac{1}{K} \sum_{k=1}^{K} \mathbf{Y_k} \mathbf{Y_k}^H \right\} \mathbf{m}(\Phi)$$

- (estimate of) the correlation matrix $\mathbf{R}$

$$\mathbf{R} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{Y_k} \mathbf{Y_k}^H$$

Beamformer (examples)

- The pratical examples are performed taking into account the following array configurations:



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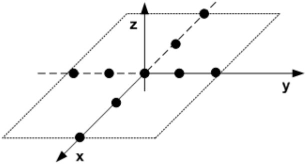
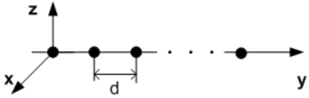
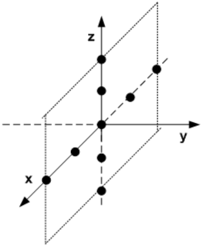
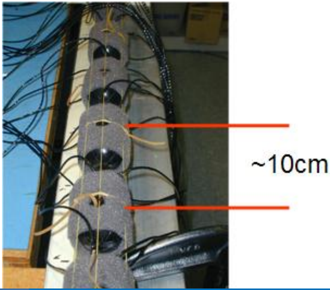






Beamformer - Vector Sensor Array (VSA)

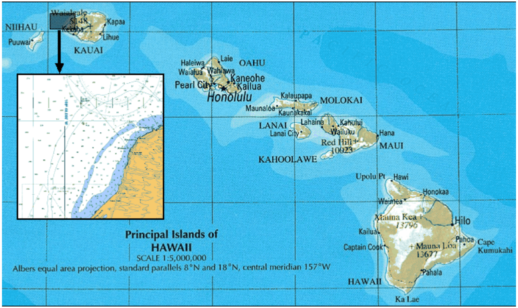
- Vector sensors emerged in 1980s as a potential alternative to omni directional hydrophones:
  - Advantages of vector sensor arrays (VSA) → Spatial filtering capabilities verified for DOA estimation;
  - High directivity systems in contrast to the omni directionality of traditional hydrophones;
  - The directional information obtained from a vector sensor allows for high performance small aperture VSA;
- Each element: 1 omni-directional hydrophone & 3 accelerometers in a tri-axial configuration;
- TV-001 type sensors, Wilcoxin Research Inc.

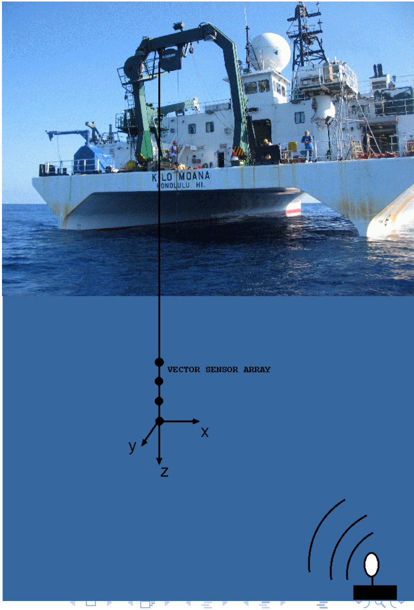


Beamformer with Experimental Data

Makai Experiment 2005:

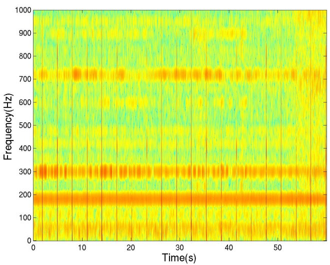
- Data acquired → North West coast of Kauai I., Hawaii, from 15 September to 2 October, 2005.
- Short vertical array of 4 vector sensors.



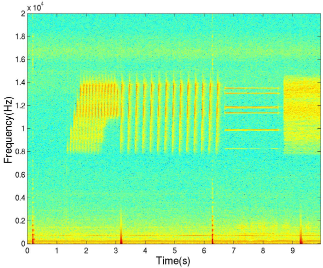


Beamformer with Experimental Data

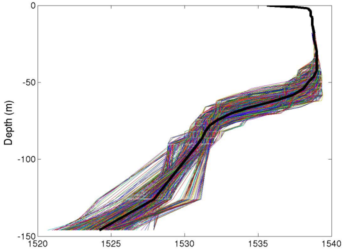
Low Frequency Signal



Signal emitted by source TB2

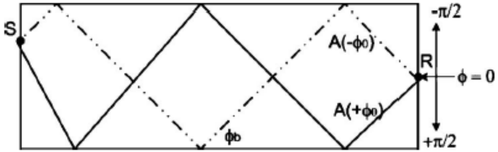


Sound speed profile during MakaiEX’05



Interferometry

- determine the bottom structure
- estimate the reflection coefficients $\mathcal{R}_b$
  - ratio between downward and upward beam

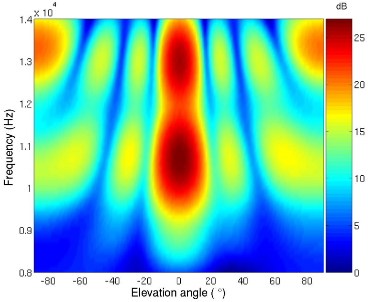


$$\mathcal{R}_b(\theta_b(\theta_0)) = \frac{B(-\theta_0)}{B(+\theta_0)}$$

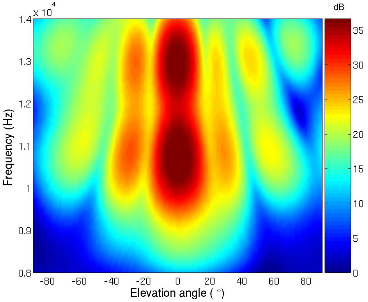
- active or passive method
- needs arrays with a large number of hydrophones (resolution)
- alternative: use vector sensors

- The beamformer response for a given azimuth diretion (MakaiEx'05 data).

Pressure

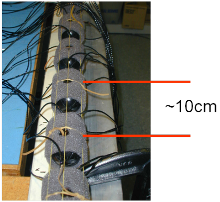


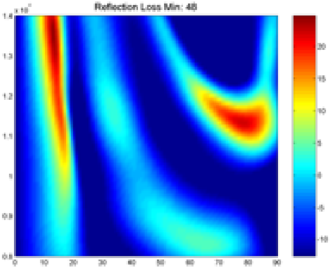
VSA

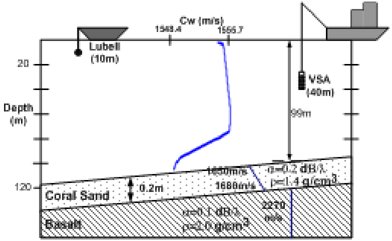


Example of seabed characterization[1]

- Data acquired during Makai Experiment, 2005
- LFM (8-14 kHz)







[1]Santos et al.,Seabed geoacoustic characterization with a vector sensor array, Journal of the Acoust. Soc. of America, Vol. 128, No. 5, Nov. 2010

Generalized beamformer

- Beam power is a function of the (search) parameter Φ

$$B(\Phi) = \mathbf{m}(\Phi)^H \mathbf{R} \mathbf{m}(\Phi)$$

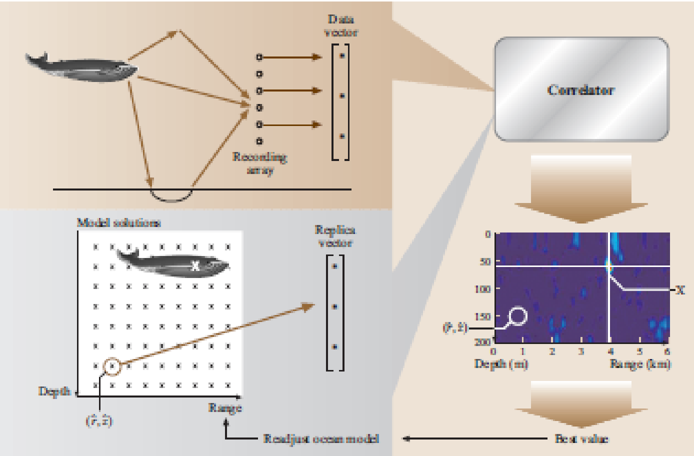
  - $\mathbf{R}$ is the correlation matrix estimated from **data**
  - Φ is the **parameter** (the direction of arrival in the previous case)
  - $\mathbf{m}(\Phi)$ is the **replica**, computed by a model (the plane-wave model in the previous model) given the parameter Φ,

But ...

- the replica $\mathbf{m}(\Phi)$ could be computed by other propagation model (ray tracing, normal modes, ...);
- the search parameter ($\boldsymbol{\Phi}$) could be the source range, source depth, c(z), or even bottom parameters....
- $B(\boldsymbol{\Phi})$ quantifies the match between the observed (field) data and the model.

Matched field processing (MFP) concept

- MFP scheme for 2D localization[2]



[2]picture from Kuperman W., Roux P., Underwater Acoustics in Springer Handbook of Acoustics, 2007

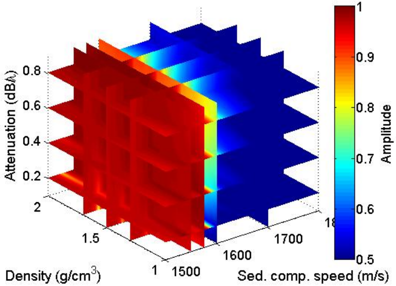
MFP issues

- the search space $\oplus$: parameter bounds, parameter discretization, use of *a priori* information;
- selection of the propagation model and input the environmental information;
- selection of the processor: behavior in presence of noisy measurements and model uncertainties, resolution - ability to distinguish between "close" replicas;
- the search algorithm: find the solution efficiently, related to the dimensionality of the search space $\oplus$ and the processor.

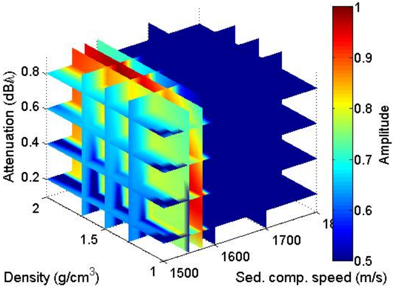
MakaiEx'05 Geoacoustic Inversion (1)

- The search parameters, in this case, are: α_p, ρ and c_p
- Output response of the processor $B(\alpha_p, \rho, c_p)$

p-only



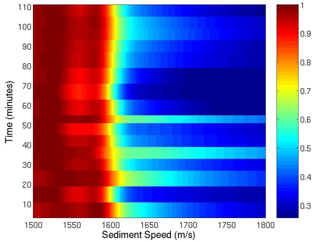
VSA



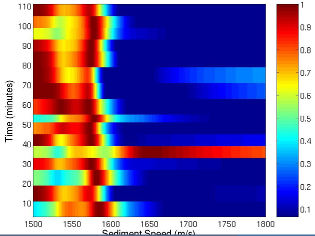
MakaiEx'05 Geoacoustic Inversion (2)

- Sediment Compressional Speed (c_p) estimation (f =13078 Hz).

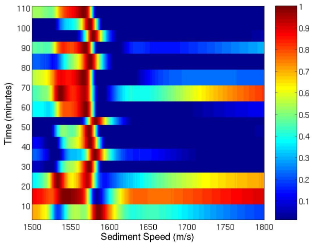
p-only



VSA



v_z-only



$$\rho = 1.35\,\mathrm{g/cm^3}$$
and
$$\alpha_p = 0.5\,\mathrm{dB}/\lambda$$

References

- Kuperman W., Roux P., Underwater Acoustics in Springer Handbook of Acoustics, 2007.
- Kuperman W. and Lynch J. , Shallow Water Acoustics, Physics Today, American Institute of Physics, Ocotber 2004.
- P.Felisberto, S.M. Jesus, Towed-array beamforming during ship's noeuvring, IEE Proc.-Radar, Sonar Navig., Vol. 143, No. 3, June 1996
- Richard 0. Nielsen, Sonarl Signal Processing, Artech House, Boston, 1991
- Santos et al.,Seabed geoacoustic characterization with a vector sensor array, Journal of the Acoust. Soc. of America, Vol. 128, No. 5, Nov. 2010

Thank you for your attention!

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